A NOTE ABOUT OPTIMAL CONTROL THEORY AND ITS APPLICATIONS TO LIFE SCIENCES

NOTA ASUPRA TEORIEI CONTROLULUI OPTIMAL ŞI APLICAȚIILE ACESTEIA ÎN ŞTIINȚELE NATURII

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Abstract. *This paper underlines some aspects of optimal control theory and the possibilities to apply this theory to solve problems from real world. The applications that we stop at are in the field of growing plants and the study of hemorrhage.*

Rezumat. *Prezenta lucrare îşi propune să evidențieze câteva din posibilitățile oferite de teoria controlului optimal pentru rezolvarea unor probleme din lumea înconjurătoare. Se va avea în vedere aplicarea teoriei controlului optimal pentru studiul creşterii plantelor şi studiul hemoragiilor sanguine.*

The main idea of this note is to show how we can control the systems that we model and how we can reach and keep some target for the system.

Control theory originated around 150 years ago when the performance of mechanical governors started to be analyzed in a mathematical way. Today in addition to technical applications of Control Theory new ones of economical and biological nature have been added.

MATERIAL AND METHOD

Control theory deals with the basic principles underlying the analysis and design of control systems. To control an object means to influence its behavior so as to achieve a desired goal. Roughly speaking there have been two main directions of work in control theory, but they are in fact complementary. One of these directions is based on the idea that a good model of the object to be controlled is available and someone wants to optimize its behavior. The techniques are closely related to the classical calculus of variations and to other areas of optimization theory; the end result is typically a preprogrammed plan. The other main direction is based on the constraints imposed by uncertainty about the model or about the environment in which the object operates. The central tool here is to use the feedback in order to correct for deviation from the desired behavior.

The basic concepts and definition for the optimal control theory are the following:

The dynamical system that is described by the state equations:

$$\dot{x}(t) = f(x(t), u(t), t), \qquad x(0) = x_0$$

where *x(t)* is state variable, *u(t)* is control variable.

The control aim is to minimize or maximize the cost functional

$$J(u(t)) = \int_0^t F(x(s), u(s), s)ds + S(x(t), t).$$

Usually the control variable *u(t)* will be constrained as follows

$$u(t) \in \Omega(t), \quad t \in [0, T].$$

RESULTS AND DISCUSSIONS

We use the optimal control theory to study the plant growing and the hemorrhage in human body.

Plant growing

A farmer wants to grow technical plants. Plants must have a specified length at some time. Using artificial light can accelerate the natural growing rate. We note with *x(t)* the function that gives the plant height at time *t*. The process can be modeled using the growing rate for plants,

$$\frac{dx}{dt} = 1 + u(t)$$

where *u(t)* represent the growth due to artificial light. We assume that at the initial moment t=0 the growing rate is 0 and the farmer wants a two units growth after one unit time interval, that means:

$$x(0) = 0, \ \mathrm{x}(1) = 2.$$

The additional growth produce supplementary production costs, these cost must be minimized. The cost functional is given by

$$J(u) = \frac{1}{2}\int_0^1 u^2(t)dt.$$

We have determined control function *u(t)*, that will be included in the solution for differential equation. Solution of the differential equations respects the condition imposed to *x(t)* and will minimize the cost function. Integrating the differential equation between *0* and *t*, and taking into account conditions for *x* we have obtained

$$\int_0^1 u(s)ds = 1.$$

On the other hand we have minimized the costs functional and we have obtained

$$J(u) = \int_0^1 \frac{1}{2}(u(s) - 1)^2 ds + \frac{1}{2},$$

The cost functional is minimum when the control takes value 1, *u(t)=1;* after that we have solved the differential equation and we have obtained the growth function *x(t)=2t.*

The study of hemorrhage in human body.

More than 4.9 million patients in the US receive transfusion every year because of loss of blood (haemorrhage) in different situations such as accidents, transplants, cancer therapies, anaemia, and others. For blood-transfusion is a need to characterize and understand how the control systems work on the cardiovascular (CVS) and respiratory systems when the blood volume change. The CVS function is transporting and exchanging substances between the environment and the cells that function in tissues.

Hemorrhagic shock is the term used for the medical syndrome resulting from decreased blood and oxygen perfusion of important organs as a consequence of a significant loss of blood volume to the circulatory system. The initial reaction to blood loss is a drop in stroke volume (blood pumped by the heart in one beat) as a result of the Frank Starling mechanism and reduced filling pressure of the heart. This results in a drop in arterial pressure and (depending on the degree and duration of blood loss) may lead to a negative and worsening spiral of reactions of CVS if not compensated for by the CVS control system. The focus of this study involves modelling CVS behaviour, its control systems and in particular the control system response to hemorrhage. There are many physiological compensatory mechanisms available to stabilize the CVS consist of a number of control loops. In this study we considered only baroreceptor reflexes consisting of high-pressure sensors responding to arterial blood pressure P_$_{as}$ and cardiopulmonary low-pressure sensors that respond to central venous pressure.

The basic model we work with models the function of the normal CVS. Mathematically the problem is formulated as a control problem. We modelled the fundamental mechanism governing the behavior for CVS in terms of changing volume. The CVS has 2 circuits: systemic and pulmonary. The left and right hearts connects these circuits. Each circuit has 2 compartments: venous and arterial. The control system consists of three major parts: the sensory portion, the central nervous system and the motor portion. Most of the control systems are negative feedback systems. Blood pressure is regulated by means of the baroreceptor reflex. For our model the stabilizing quantities are: the heart frequency, the heart muscle contractility, the venous tone, the arterial resistance, the ventilation rate, the blood volume. The stabilized quantities are: the blood pressure, the cardiac output, the oxygen and the carbon dioxide. The next equations give the CVS model without hemorrhage.

$$c_{as}\dot{P}_{as} = Q_l - F_s,$$

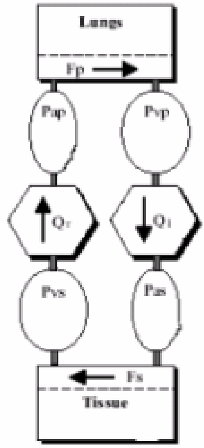
$$c_{vs}\dot{P}_{vs} = F_s - Q_r,$$

$$c_{vp}\dot{P}_{vp} = F_p - Q_l,$$

$$c_{as}\dot{P}_{as} = Q_l - F_s,$$

$$\ddot{S}_l = -\gamma_l\dot{S}_l - \alpha_l S_l + \beta_l H,$$

$$\ddot{S}_r = -\gamma_r\dot{S}_r - \alpha_r S_r + \beta_r H.$$



The first four equations are obtained by the compartment mass balance relations i.e. the change in volume is the difference between inflow and outflow. There are essentially two possibilities for the heart to change cardiac output: heart rate and contractility. These are connected by Bowditch effect. In mathematical terms this effect is given by the last 2 equation of the model. We have considered a linear relation between volume and pressure.

In the previous differential equation system we will include the term in the first equation corresponds to the blood loss. The term corresponds to the transfusion is added in the second equation. The system that model the arterial hemorrhage and the venous transfusion is given by the previous one but the first tow equations are

$$c_{as}\dot{P}_{as} = Q_l - F_s - \frac{R_0}{P_{asi}} P_{as},$$

$$c_{vs}\dot{P}_{vs} = F_s - Q_r + \frac{1}{R_{transf}}(P_h - P_{vs}).$$

We have controlled the system and stabilize it at the equilibrium point. To reach this goal we control the heart rate:

$$\dot{H} = u,$$

and the cost functional:

$$J(u) = \int_0^t \Big(q_1 (P_{as}(s) - \overline{P}_{as})^2 + q_2 u^2(s) \Big) ds,$$

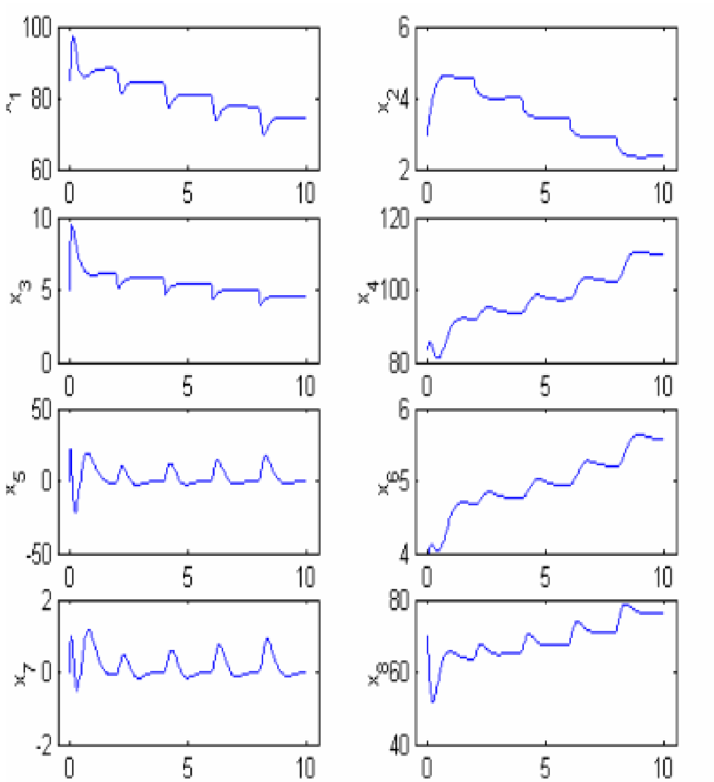
where $q_1 > q_2$ are weights. We use weights to penalize *u,* which is the heart rate variation in order to avoid non-physiological controls and to enforce efficient operations. For the model accuracy we choose $q_1 > q_2$.

For the computational part we used Matlab software. The algorithm of the program follows the next steps.

- Find the initial steady-state (xe) and goal steady-state (xg) with one fixed variable such as *Pas*.
- Linearize the system around the goal steady state.
- Find the solution of the algebraic Riccati equation corresponding to the linear quadratic problem for the liberalized system that provides the feedback control for the linear system which is also applied to the nonlinear system.
- Simulate the system

$$\dot{x}(t) = f(x(t), p(t)) + Bu(t)$$

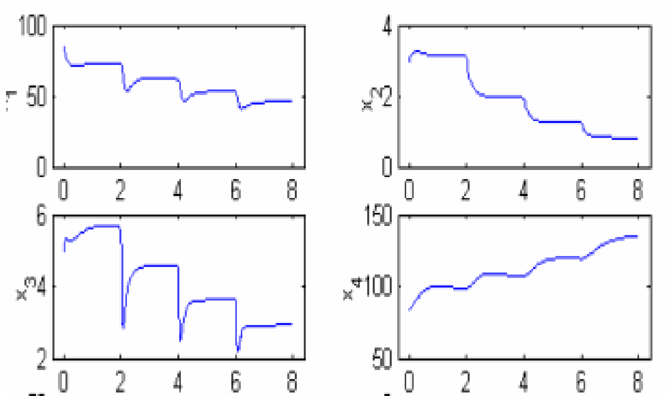
The simulation for volume decreasing (step change in volume) is given in Figure 1

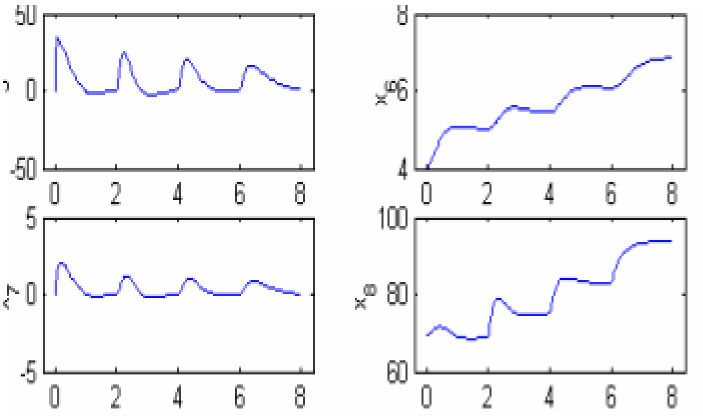


$x_1 = P_{as}$ $x_2 = P_{vs}$ $x_3 = P_{vp}$ $x_4 = S_l$ $x_5 = \sigma_l$ $x_6 = S_r$ $x_7 = \sigma_r$ $x_8 = H$

Figure 1 - Simulation for volume decreasing

This model was extended from the basic model, and we want to see how a decreasing volume will affect the model. We considered step changes in volume to see what would happen. The pressure goes down as well as the heart rate goes up. The contractility also goes up (by Bowditch) as compensation for reduced arterial pressure.





$x_1 = P_{as}$ $x_2 = P_{vs}$ $x_3 = P_{vp}$ $x_4 = S_l$ $x_5 = \sigma_l$ $x_6 = S_r$ $x_7 = \sigma_r$ $x_8 = H$.

Figure 2 - Simulation for blood loss.

We see that the behaviour for the pressure and the heart rate stay the same as in case without hemorrhage, but change at a slower rate. The numerical result for blood loss simulation seems to be unrealistic, because we choose our volume change as a step function. We must choose this as a continuous function, so the result will be smoother.

CONCLUSIONS

The optimal control theory has many applications in life sciences, as we have seen from the previous examples. It is up to us if the modelling part simplifies the reality too much, but is necessary to make a compromise between reality and modelling. Both examples that we have considered can be improved but with these difficulties the models are realistic and have gave good results.

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